- ECE 265 -

ANALYTIC TECHNIQUES IN ELECTRONIC AND COMMUNICATION ENGINEERING

LAB #8: Z-TRANSFORM

Definition z-transform

Definition 9.1: z-transform Given the sequence $\{x_n = x[n]\}_{n=0}^{\infty}$, the z-transform is defined as follows:

$$X(z) = \mathfrak{Z}[\{x_n\}_{n=0}^{\infty}] = \sum_{n=0}^{\infty} x_n z^{-n} = \sum_{n=0}^{\infty} x[n] z^{-n}, \qquad (9\text{-}1)$$

which is a series involving powers of $\frac{1}{z}$.

E.g.

a) Find the z-transform of discrete unit step function of u(n) analytically.

b) Find the z-transform of the sequence x(n)=exp(an) analytically.

c) Find the z-transform of sequence x(n)=cos(an) analytically.

d) By using Matlab compute all the z-transform of sequences given in table 9.1.

   a.

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>> syms n

>> X=ztrans(1/4^n)

X =

4*z/(4*z-1)
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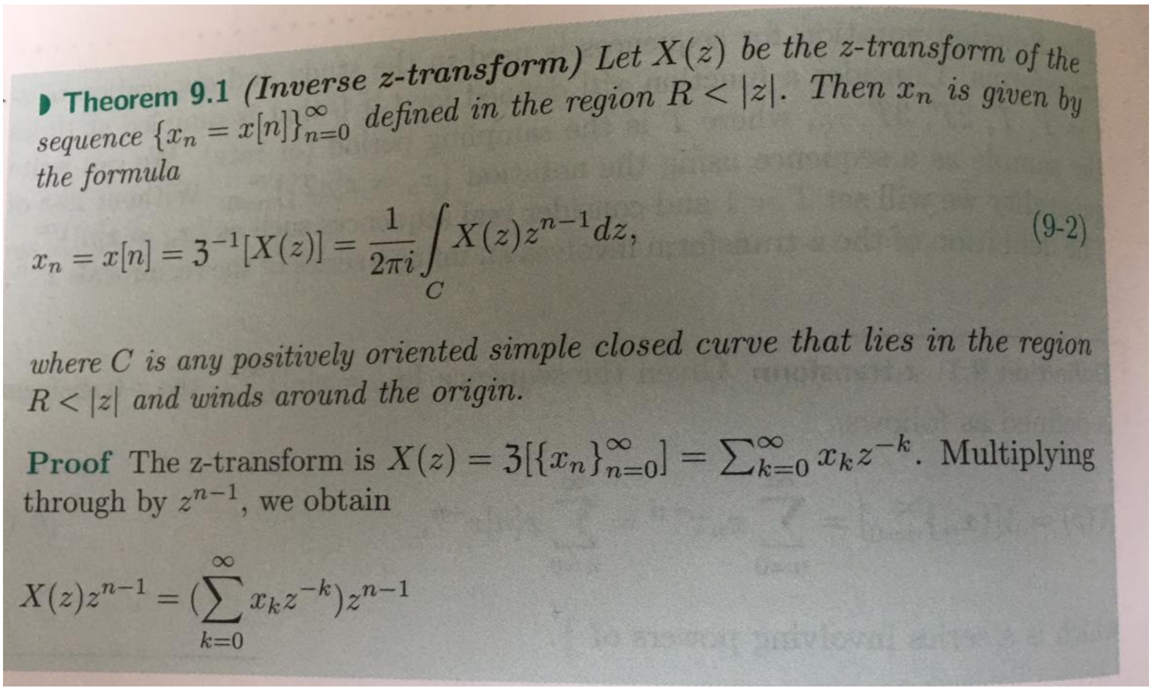
	Sequence	z-transform
1	$\delta[n]$	1
2	$u[n]$	$\frac{z}{z-1}$
3	b^n	$\frac{z}{z-b}$
4	$b^{n-1}u[n-1]$	$\frac{1}{z-b}$
5	e^{an}	$\frac{z}{z-e^a}$
6	n	$\frac{z}{(z-1)^2}$
7	n^2	$\frac{z(z+1)}{(z-1)^3}$
8	nb^n	$\frac{bz}{(z-b)^2}$
9	ne^{an}	$\frac{ze^a}{(z-e^a)^2}$
10	$\sin(an)$	$\frac{i\left(-1+e^{2ia}\right)z}{2(e^{ia}-z)(-1+e^{ia}z)}$
11	$b^n \sin(an)$	$\frac{ib\left(-1+e^{2ia}\right)z}{2(be^{ia}-z)(-b+e^{ia}z)}$
12	$\cos(an)$	$\frac{z\left(1+e^{2ia}-2e^{ia}z\right)}{2(e^{ia}-z)(-1+e^{ia}z)}$
13	$b^n \cos(an)$	$\frac{z\left(b+be^{2ia}-2e^{ia}z\right)}{2(be^{ia}-z)(-b+e^{ia}z)}$

Table 9.1 z-transforms of some common seque

Definition of inverse z-transform



Theorem 9.1 *(Inverse z-transform) Let $X(z)$ be the z-transform of the sequence $\{x_n = x[n]\}_{n=0}^{\infty}$ defined in the region $R < |z|$. Then x_n is given by the formula*

$$x_n = x[n] = \mathfrak{Z}^{-1}[X(z)] = \frac{1}{2\pi i}\int_C X(z) z^{n-1} dz, \quad (9\text{-}2)$$

where C is any positively oriented simple closed curve that lies in the region $R < |z|$ and winds around the origin.

Proof The z-transform is $X(z) = \mathfrak{Z}[\{x_n\}_{n=0}^{\infty}] = \sum_{k=0}^{\infty} x_k z^{-k}$. Multiplying through by z^{n-1}, we obtain

$$X(z)z^{n-1} = \left(\sum_{k=0}^{\infty} x_k z^{-k}\right) z^{n-1}$$

e) Obtain the inverse z-transform of X(z)=2z/(2z-1) analytically.

f) On matlab use iztrans

g) By using residue theorems of inverse z-transform.

$x_n = Z^{-1}[X(z)] = \sum_{j=1}^{k} Res[X(z)z^{n-1}, z_j]$

$x_n = Z^{-1}[X(z)] = \sum_{j=1}^{k} \lim_{z-z_i}(z-z_i)X(z)z^{n-1}$

Homework: Find xn of given $X(z)=1/(1-\sqrt{2}/z+z^{-2})$ both analytically and on matab.

GOOD LUCK !!!